

Name: _____

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. Let t be the number of days it takes for one of Justin Bieber’s tattoos to heal. Assume that $c > 0$ and the probability density function for t is

$$p(t) = \begin{cases} c(1 - 0.025t) & \text{for } 0 \leq t \leq 40, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Sketch $p(t)$ on the interval $-10 \leq t \leq 50$. What value of c makes $p(t)$ into a probability density? Use this value of c to answer the following questions.
- (b) What is the likelihood that a random tattoo will heal within 10 days?
- (c) How long does it take on average for Justin’s tattoos to heal?
- (d) If Justin goes wild and gets lots of tattoos at once, how many days will it take for half of them to heal?

Hint: Find the median of $p(t)$, i.e. a number M such that the vertical line $t = M$ splits the area under $p(t)$ exactly in half.

2. A patient takes 500 milligrams (mg) of a drug each day and it is removed from the body at a rate of 70% of the amount in the body per day. Let y be the amount of drug and t be the time in days.

- (a) Write a differential equation describing the dynamics of the amount of drug in the body: $\frac{dy}{dt}$.
- (b) Calculate the equilibrium value(s).
- (c) Determine the stability of the equilibrium value(s).
- (d) Explain the meaning of your answers in parts (b) and (c) using a complete sentence and including units.

3. Use separation of variables to solve the differential equation $\frac{dB}{dt} = \frac{3B}{t}$ subject to the initial condition $B(0) = 4$. You may assume $t \geq 0$, $B(t) > 0$.

1	2	3	total (30)