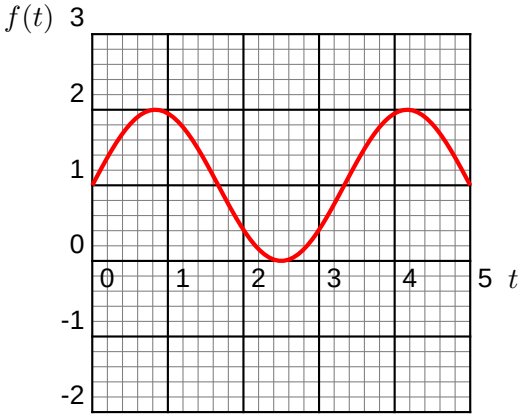
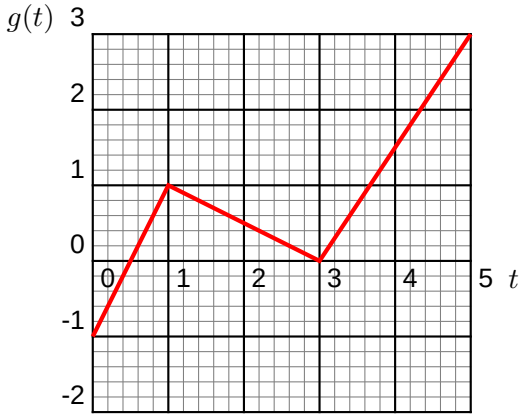
Name: _____

Please show all work. Supply brief narration with your solutions and draw conclusions (with units).

1. To test Justin Bieber's clearance rate for Exasperin, he is given a bolus injection of 50 milligrams (mg) of Exasperin. After 2 hours Justin's level of Exasperin $E(2)$ is 30 mg. Assuming Exasperin is cleared proportionally, we get exponential decay $E(t) = E(0)e^{-rt}$. What is the relative hourly clearance rate r? What is the half life of Exasperin?

2. Graphs of $f(t)$ and $g(t)$ are given below.





   (a) What is the average rate of change of g on the interval $0 \le t \le 5$? On the same grid as g draw a straight line with the same average rate. What is the average rate of change of f on the interval $3 \le t \le 5$?

   (b) Estimate the instantaneous rate of change of f at $t = 3$ using $\Delta t = 0.1$.

   (c) Graphically differentiate f and g on the same grids.

3. After a few experiments, Justin Bieber's voice coach is able to model how well Justin stays in tune (stars) depending on the dosage of Exasperin (x mg) with the function

$$f(x) = \frac{6}{5} + 54000\frac{2x + 15}{4500 + x^2}$$

   (a) What's Justin's star rating without Exasperin? What is it for very large doses of Exasperin? Sketch $f(x)$ on the interval $0 \le x \le 700$.

   (b) Determine the highest rating Justin can achieve and what dosage of Exasperin is needed to make it so.

(CONTINUED ON THE OTHER SIDE)

1	2	3	4	5	6	7	total (70)

(CONTINUED FROM THE OTHER SIDE)

4. The 4 meter wide moat around Justin Bieber's house has a cross section shaped like the graph of $y = 4 - x^2$, $-2 \le x \le 2$.

   (a) Obtain bounds for the area from the middle of the moat to the shore (by symmetry this is half the total cross sectional area) using upper and lower estimates with two subintervals of $0 \le x \le 2$. Sketch. What can you conclude about the exact area from your estimates?

   (b) Do the same with trapezoidal (average of left and right) and midpoint (the height of the rectangle is the value of y at the midpoint of a subinterval) estimates. Sketch.

   (c) Find Simpson's estimate (weighted average of midpoint and trapezoidal: $\frac{2}{3}$ midpoint $+\frac{1}{3}$ trapezoidal). Compare it to the exact area by expressing the latter as a definite integral and evaluating it using the Fundamental Theorem of Calculus.

5. Find antiderivatives (indefinite integrals) for the following functions. Show work.

$$\text{(a) } \frac{t}{1+t^2} \qquad \text{(b) } 3^{5t+1} \qquad \text{(c) } t^3 \cos(t)$$

6. Let t be the number of days it takes for one of Justin Bieber's tattoos to heal. Assume that $c > 0$ and the probability density function for t is

$$p(t) = \begin{cases} c\,(1 - 0.025\,t) & \text{for } 0 \le t \le 40, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Sketch $p(t)$ on the interval $-10 \le t \le 50$. What value of c makes $p(t)$ into a probability density? Use this value of c to answer the following questions.

   (b) What is the likelihood that a random tattoo will heal within 8 days?

   (c) How long does it take on average for Justin's tattoos to heal?

   (d) If Justin goes wild and gets lots of tattoos at once, how many days will it take for half of them to heal?

7. Justin Bieber is placed on a drip delivering d milligrams of Exasperin per hour (mg/h).

   (a) Assuming Exasperin is cleared proportionally at a rate r mg/h, find an autonomous differential equation for the level of Exasperin in Justin $E(t)$ by expressing the total rate dE/dt in terms of drip rate and clearance rate.

   (b) Find all equilibria for this differential equation in terms of d and r. Sketch dE/dt as a function of E (pick some representative values of d and r for the purposes of sketching) to determine stability of each equilibrium. What is the long term prediction for Justin's level of Exasperin? Express your answer in terms of d and r.

   (c) Justin is starting to record a new album and the latest model says he needs a level of 50 mg of Exasperin in him to stay in tune. An experiment similar to one in problem 1 is performed and this time the relative hourly clearance rate r is found to be 0.2. At what rate d should Exasperin be drip delivered to provide for Justin's need?