

Name: _____

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. A researcher starts a bacterial culture in a petri dish. Two days later the colony is 4 million strong. The day after that it reaches 6 million. Assuming the growth is exponential, what will the size be on the fourth day?
2. The level of medication for a while varies according to $s(t) = 15 + t^2 - t$ where time t is measured in days. Compute the derivative of s with the respect to t using the definition of derivative. Find and illustrate on a graph
 - (a) Initial level and after 3 days.
 - (b) The instantaneous rates of change at those two times.
 - (c) The average rate of change during that period of time.
 - (d) The equation of the tangent line at $t = 3$.
3. A population of wasps x_t satisfies the recursion $x_{t+1} = 2\sqrt{x_t}$. Find fixed points of the recursion (equilibria) and do some cobwebbing on a graph or numerical experimentation to determine their stability (attracting vs. repelling). Describe what happens to the population in the long run, if $x_0 = 0$. Same, if $x_0 = 1$.
4. Find y' where

(a) $y = x^3 \ln x$ (b) $y = \frac{x^2}{\cos(5x)}$ (c) $y = x^{e^x}$ (d) $\sin(3y) + \exp(2x) = y^2$

5. The windpipe contracts during a cough from the rest radius R to radius $r \leq R$. The speed of the exiting is $v = a(R - r)r^2$, where a is a positive constant. What value of r maximizes speed?
6. Evaluate the following limits. Justify your answers. If you use l'Hôpital's rule, be sure to specify which case.

(a) $\lim_{x \rightarrow \infty} \frac{x}{4x + 1}$ (b) $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{1}{x}\right)$ (c) $\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$ (d) $\lim_{x \rightarrow 0^+} x^3 \ln x$

7. Find antiderivatives for the following functions.

(a) $\sin(3x)$ (b) $x \sin(3x^2)$ (c) $x^2 \sin(3x)$ (d) $\ln x$ (hint: by parts)

8. After an initial bolus injection of 25 mg of a drug, a patient is placed on a drip delivering 1.4 mg per hour. If the drug is cleared by the patient at a rate of $0.9 - 0.05t^4$ mg/h as a function of time, what is the amount of drug in the patient 3 hours later?

1	2	3	4	5	6	7	8	total (80)	%