

Name: \_\_\_\_\_

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. A researcher starts a bacterial culture in a petri dish. A day later the colony is 40 million strong. The next day it reaches 60 million. Assuming the growth is exponential, what was the initial size?
2. The level of a hormone varies according to  $s(t) = 4 + 2 \cos(0.4t)$  where time  $t$  is measured in months. Find and illustrate on a graph
  - (a) Initial size and the size after a month.
  - (b) The instantaneous rates of change at those two times.
  - (c) The average rate of change during that period of time.

3. Find the derivatives of

(a)  $t3^{2t}$       (b)  $\frac{\ln t}{\sqrt{t}}$

4. Find the second derivative of  $f(t) = \frac{1}{t^2 + 1}$  and use it to describe the curvature of the graph of  $f$  for  $t \geq 0$ .

5. A population  $x_t$  has *per capita* production  $\frac{2}{x_t + 1}$ . Write down the discrete dynamical system for  $x_t$ . Find equilibria and do some cobwebbing on a graph to determine their stability. Find the derivative of the updating function. What are its values at each equilibrium? Describe in words what happens in the long run.

Hint:  $x_{t+1} = f(x_t)$ , where the updating function  $f$  is the per capita production times the size.

1	2	3	4	5	total (50)	%

Prelim. course grade:                      %