

Name: _____

Please show all work and justify your answers.

1. Solve the recurrence $x_n = x_{n-1} + 6x_{n-2}$ subject to initial conditions $x_0 = 2, x_1 = 1$. Check your solution by computing x_2 and x_3 using the recurrence relation and comparing the values of x_0, x_1, x_2, x_3 from your formula.
2. Consider the linear system $x = 1 - 2y - 3z, 5y = 1 - 4x - 6z, 9z = 1 - 7x - 8y$.
 - (a) Find the augmented matrix A for this system.
 - (b) Use Gauss-Jordan elimination to find $\text{rref}(A)$. Show steps.
 - (c) Find a parametric formula for the solution set. Sketch and describe the solution set.
3. Let L be the line $y = -3x$ in the plane. Find matrices A such that $\mathbf{x} \mapsto A\mathbf{x}$ is
 - (a) orthogonal projection of \mathbf{x} to L ,
 - (b) reflection of \mathbf{x} with respect to L .

In each case compute A^2 and briefly explain your result geometrically.

4. Let $A = \begin{bmatrix} -7 & 3 \\ -18 & 8 \end{bmatrix}$.
 - (a) Find the eigenvalues of A and corresponding eigenvectors.
 - (b) Let S be the matrix whose columns are eigenvectors of A . Verify that $S^{-1}AS$ is diagonal with entries the eigenvalues of A .
 - (c) Sketch the eigenspaces and give a geometrical description of the linear map $\mathbf{x} \mapsto A\mathbf{x}$.

1	2	3	4	total (40)