Midterm 2 / 2014.11.7 / CS 3333.001 / Mathematical foundations of computer science

Name: ____

Please show all work and justify your answers.

1. Show that for $n > 0$ the sum of consecutive squares has a closed form expression, by proving by induction that
$$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$$

2. Consider the linear system $2x + 3y + 5 = 0$, $x + y + z = 5$.
   (a) What augmented matrix A represents this system? Use Gauss-Jordan elimination to find its reduced row echelon form. Show steps.
   (b) Use (a) to find all solutions to the system in terms of the free variable(s).
   (c) Check your answer.
   (d) Sketch the solution set.

3. For each of the following parts give a concrete example of a 2×2 real matrix A satisfying the given conditions.
   (a) A not upper triangular, $\det(A) = 0$
   (b) A not diagonal, $\det(A) \neq 0$
   (c) A not diagonal with 2 distinct eigenvalues
   (d) A with no real eigenvalues

4. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A and its roots — the eigenvalues of A.
   (b) For each of the eigenvalues you found in (a) find corresponding eigenvectors.
   (c) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. Check your answer.
   (d) Sketch the eigenspaces and describe the geometrical effects of the plane transformation $x \mapsto Ax$.

1	2	3	4	total (40)