Midterm 2 / 2013.11.26 / CS 3333.001 / Mathematical foundations of computer science

Name: ______________________________

Please show all work and justify your answers.

1. (a) Convert to matrix notation the system of linear equations $x_1 + 3x_2 + 2x_3 = 1$, $x_2 + x_3 = -2x_1$, $3x_3 + 4x_1 = -3$.
   (b) Use Gauss-Jordan elimination to solve this system.
   (c) Draw a conclusion about the determinant of the coefficient matrix. Explain.

2. Let $A = \begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find corresponding eigenvectors.
   (c) Sketch the eigenspaces. In a few words give a geometric description of the linear transformation of the plane $v \mapsto Av$.

3. (a) A dozen male pilgrims queue up at the latrine. How many different possible queues?
   (b) They wash their hands and return to the picnic area, where they are seated at a round table. How many different neighbor arrangements?
   (c) Three female pilgrims join the Thanksgiving feast. How many neighbor arrangements if no two ladies want to sit next to each other?
   (d) For bonus points generalize to find a closed form expression for m ladies and n gentlemen $(m \leq n)$.
   Hint: $P(r, k) = \dfrac{r!}{(r-k)!}$.

4. What's the probability of rolling a seven with three dice?

   Hint: Think of rolling a die as picking that many of particular type of object. Keep in mind that you must pick at least one object of each of the 3 types. Eventually you want to pick 7 objects. Meanwhile, k-combinations out of n with repeats: $C(n + k - 1, k)$, where $C(r, k) = P(r, k)/k!$.

5. Every day Fred has a pint on the way home from work in one of two pubs. Saucy Penguin is less of a detour, so Fred is twice as likely to stop there than at the Liquid Office. Ale is really fresh at the Penguin so Fred is three times as likely to order that than his only other choice: lager. At the Office he is as likely to order one as the other. If Fred comes home with a lager under his belt, what are the chances he stopped by the Office?

   Hint: Bayes Theorem for two events in a sample space: $p(F|E) = \dfrac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\overline{F})p(\overline{F})}$.

6. Express the probability of being within ± 1 of the mean in terms of the error function, if the probability density is normal with mean 5 and standard deviation 2.

   Hint: $p(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Make the substitution $z = \dfrac{x - \mu}{\sigma}$ and use the fact that for the standard normal distribution $(\mu = 0, \sigma = 1)$ we have $P(0 < z < Z) = \dfrac{1}{2}\operatorname{erf}\left(\dfrac{Z}{\sqrt{2}}\right)$.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %