Midterm 1 / 2013.10.10 / CS 3333.001 / Mathematical foundations of computer science

Name: ______________________________

Please show all work and justify your answers.

1. Compute the following.
   (a) Hexadecimal expansion of 255.
   (b) Binary expansion of 255.
   (c) Decimal of hexadecimal AAA.
2. Apply Euclid's algorithm to 56 and 25 to show that they are co-prime. Find the Bézout coefficients.
3. Suppose $m \geq 2$. Show that if $a \equiv a' \bmod m$ and $b \equiv b' \bmod m$, then $ab \equiv a'b' \bmod m$.

   Hint: $ab - a'b' = ab - a'b + a'b - a'b'$
4. Use the Chinese remainder theorem to solve the following system of congruences:

$$x \equiv 3 \bmod 5, \qquad 3x \equiv 5 \bmod 7, \qquad 3x \equiv 4 \bmod 11.$$

   Hint: First eliminate the leading coefficient by finding its multiplicative inverse.
5. Prove by induction that $n! < n^n$ for $n > 1$.
6. Suppose f is a function given recursively by $f(0) = 3$ and $f(n) = -2f(n-1)$ for $n \geq 1$. Find a formula for f and prove its validity by induction.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %