Final / 2013.12.12 / CS 3333.001 / Mathematical foundations of computer science

Name: ____________________

Please show all work and justify your answers.

1. Apply Euclid's algorithm to 70 and 27 to show that they are co-prime. Find the Bézout coefficients.

2. Suppose $m \geq 2$. Show that if $a \equiv a' \bmod m$ and $b \equiv b' \bmod m$, then $a+b \equiv a'+b' \bmod m$.

3. Use the Chinese remainder theorem to solve the following system of congruences: $x \equiv 2 \bmod 5$, $2x \equiv 5 \bmod 7$, $3x \equiv 7 \bmod 11$.

4. For which integers $n \geq 0$ is $n^2 \leq n!$? Prove your assertion.

5. Suppose f is a function given recursively by $f(0) = 2$ and $f(n) = -3f(n-1)$ for $n \geq 1$. Find a formula for f and prove its validity by induction.

6. Convert to matrix notation the system of linear equations $x_3 + 2x_2 = 4$, $x_3 + 3x_1 = 2$, $4x_1 + 3x_2 = 3$. Use Gauss-Jordan elimination to solve this system (show intermediate steps). Find the determinant of the coefficient matrix. Explain.

7. Let $A = \begin{bmatrix} 4 & 5 \\ 6 & 2 \end{bmatrix}$. Find the eigenvalues of A. Find corresponding eigenvectors. Sketch the eigenspaces. In a few words give a geometric description of the linear transformation of the plane $v \mapsto Av$.

8. Rudolph is out with a cold, so Santa harnesses his other eight reindeer in pairs (::::). How many ways can Santa harness the reindeer if he doesn't break up the pairs? Generalize to $2n$ reindeer.

9. What's the probability of rolling an eight with four dice?

   Hint: k-combinations out of n with repeats: $C(n+k-1, k)$, where $C(r,k) = \dfrac{r!}{(r-k)!k!}$.

10. Express the probability of being within ± 2 of the mean in terms of the error function, if the probability density is normal with standard deviation 3.

    Hint: $P(0 < z < Z) = \dfrac{1}{2}\,\mathrm{erf}\left(\dfrac{Z}{\sqrt{2}}\right)$, where $z = \dfrac{x-\mu}{\sigma}$.

1	2	3	4	5	6	7	8	9	10	total (100)