

Linear first order differential equations:

$$y' + py = q, y = \frac{1}{v} \int vq dx, \text{ where } v = e^{\int p dx}$$

Limits:

$$\lim_{n \rightarrow \infty} x^{\frac{1}{n}} = 1 \text{ for } x > 0$$

$$\lim_{n \rightarrow \infty} n^{\frac{1}{n}} = 1$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$$

Taylor series:

$$f(x) = \sum_{k=0}^n \frac{f^{(k)}(c)}{k!} (x-c)^k + \frac{f^{(n+1)}(c^*)}{(n+1)!} (x-c)^{n+1} \text{ for some } c^* \text{ between } c \text{ and } x$$

$$\text{Geometric series } \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\text{Binomial series } (1+x)^r = \sum_{k=0}^{\infty} \frac{r(r-1)\dots(r-k+1)}{k!} x^k$$

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Fourier series:

Given piecewise continuous $f(t)$ on an interval $[c-L, c+L]$ centered at c , at points of continuity

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right]$$

where

a_0 is the average of f on the interval and for $n \geq 1$

$$a_n = \frac{1}{L} \int_{c-L}^{c+L} f(t) \cos \frac{n\pi t}{L} dt$$

$$b_n = \frac{1}{L} \int_{c-L}^{c+L} f(t) \sin \frac{n\pi t}{L} dt$$