

## Integrals:

$$\text{Fundamental theorem: } \int_a^x f'(t) dt = f(x) - f(a) \quad \left[ \int_a^x f(t) dt \right]' = f(x) \quad \text{or } F(x) = \int f(x) dx \Leftrightarrow F'(x) = f(x)$$

$$\text{Linearity (on functions): } \int [a \cdot f(x) + b \cdot g(x)] dx = a \int f(x) dx + b \int g(x) dx$$

$$\text{Additivity (on intervals): } \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\text{Products (by parts): } \int f'(x)g(x) dx = f(x)g(x) - \int f(x)g'(x) dx \quad \text{or } \int u dv = uv - \int v du$$

$$\text{Composite functions (substitution): } \int f(u(x))u'(x) dx = \int f(u) du$$

$$\text{Powers: } \int x^r dx = \begin{cases} \frac{1}{r+1}x^{r+1} & \text{if } r \neq -1 \\ \ln|x| & \text{if } r = -1 \end{cases}$$

$$\text{Exponentials and logarithms (base } a > 0, \ln = \log_e, e = \lim_{n \rightarrow \infty} \left[ 1 + \frac{1}{n} \right]^n \approx 2.7182818284590): a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_n \quad a^{1/n} = \sqrt[n]{a}$$

$$a^{-r} = 1/a^r \quad a^{\log_a x} = x \quad \log_a(a^y) = y \quad a^{x+y} = a^x a^y \quad (a^x)^y = a^{xy} \quad \log_a(xy) = \log_a x + \log_a y \quad \log_a(x^y) = y \log_a x$$

$$(a^x)' = \ln a \cdot a^x \quad (\log_a x)' = \frac{1}{\ln a} \frac{1}{x} \quad \int a^x dx = \frac{1}{\ln a} a^x \quad \int \log_a x dx = \frac{1}{\ln a} (x \ln x - x)$$

$$\text{Hyperbolic functions: } \cosh x = \frac{1}{2}(e^x + e^{-x}) \quad \sinh x = \frac{1}{2}(e^x - e^{-x}) \quad (\cosh x)^2 - (\sinh x)^2 = 1$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \quad \sinh(x \pm y) = \sinh x \cosh y \mp \cosh x \sinh y$$

$$\cosh x \cosh y = \frac{1}{2}[\cosh(x+y) + \cosh(x-y)] \quad \sinh x \sinh y = \frac{1}{2}[\cosh(x+y) - \cosh(x-y)]$$

$$\sinh x \cosh y = \frac{1}{2}[\sinh(x+y) + \sinh(x-y)] \quad (\cosh x)^2 = \frac{1}{2}[\cosh(2x) + 1] \quad (\sinh x)^2 = \frac{1}{2}[\cosh(2x) - 1]$$

$$\tanh x = \sinh x / \cosh x \quad \coth x = \cosh x / \sinh x \quad \operatorname{sech} x = 1 / \cosh x \quad \operatorname{csch} x = 1 / \sinh x$$

$$(\sinh x)' = \cosh x \quad (\cosh x)' = \sinh x \quad (\tanh x)' = (\operatorname{sech} x)^2$$

$$(\coth x)' = -(\operatorname{csch} x)^2 \quad (\operatorname{sech} x)' = -\tanh x \operatorname{sech} x \quad (\operatorname{csch} x)' = -\coth x \operatorname{csch} x$$

$$\text{Trigonometric functions: } \cos x = \frac{1}{2}(e^{ix} + e^{-ix}) \quad \sin x = \frac{1}{2i}(e^{ix} - e^{-ix}) \quad (\cos x)^2 + (\sin x)^2 = 1$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \quad \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)] \quad \sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)] \quad (\cos x)^2 = \frac{1}{2}[1 + \cos(2x)] \quad (\sin x)^2 = \frac{1}{2}[1 - \cos(2x)]$$

$$\tan x = \sin x / \cos x \quad \cot x = \cos x / \sin x \quad \sec x = 1 / \cos x \quad \csc x = 1 / \sin x$$

$$(\sin x)' = \cos x \quad (\cos x)' = -\sin x \quad (\tan x)' = (\sec x)^2$$

$$(\cot x)' = -(\csc x)^2 \quad (\sec x)' = \tan x \sec x \quad (\csc x)' = -\cot x \csc x$$

$$\int \sec x dx = \ln |\sec u + \tan u| \quad \int \csc x dx = -\ln |\csc u + \cot u|$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \left( \frac{x}{a} \right) \quad (a > 0) \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right) \quad \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \left( \frac{x}{a} \right) \quad (x > a > 0)$$

$$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left| \frac{x}{a} \right| \quad (a > 0, x \neq 0) \quad \int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left( \frac{x}{a} \right) \quad (0 < x < a)$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left( \frac{x}{a} \right) \quad \int \frac{dx}{a^2 + x^2} = -\frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$