

Frenet frame

Parametrized curve:

Position: $x(t): [a, b] \rightarrow \mathbf{R}^3$

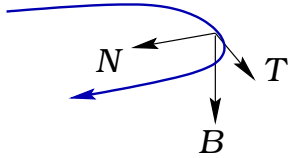
Velocity: $v(t) = x'(t)$

Acceleration: $a(t) = v'(t) = x''(t)$

Arclength:

$$s'(t) = |x'(t)| = |v(t)| \quad s(t) = \int_a^t |x'(\tau)| d\tau = \int_a^t |v(\tau)| d\tau$$

Frenet frame:



$$\left\{ \begin{array}{l} \text{Tangent:} \quad T = \frac{dx}{ds} = \frac{x'}{|x'|} = \frac{v}{|v|} \\ \text{Principal normal:} \quad N = \frac{dT/ds}{|dT/ds|} = \frac{T'}{|T'|} \\ \text{Binormal:} \quad B = T \times N \end{array} \right.$$

$$\text{Curvature and torsion: } \kappa = \left| \frac{dT}{ds} \right| = \frac{|v \times a|}{|v|^3} \quad |\tau| = \left| \frac{dB}{ds} \right|$$

$$\text{Frenet-Serret formulas: } \frac{d}{ds} \begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$