Flow past a cylinder

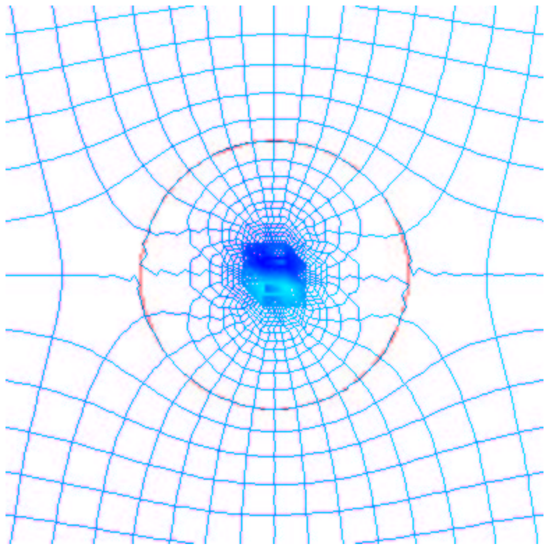
Consider an infinite cylinder with cross-section the unit circle. The Zhukovsky map $g(z) = z + 1/z$ squashes the unit circle to the interval $[-2, 2]$ (traversed both ways).

Start with a horizontal flow left to right with speed 1. Then $f(z) = z$ is a complex potential for this flow. Composing with g we obtain a complex potential for the flow past the cylinder $f(g(z)) = z + 1/z$.

We can obtain flow lines by setting the imaginary part $\operatorname{Im} f(g(x+iy)) = y - \dfrac{y}{x^2+y^2} = \text{const}\,.$

Equipotential lines are obtained by setting the real part $\operatorname{Re} f(g(x+iy)) = x + \dfrac{x}{x^2+y^2} = \text{const}\,.$

Here are contour plots of the real and imaginary parts of $f(g(z))$.



Flow past a plate with an angle of attack

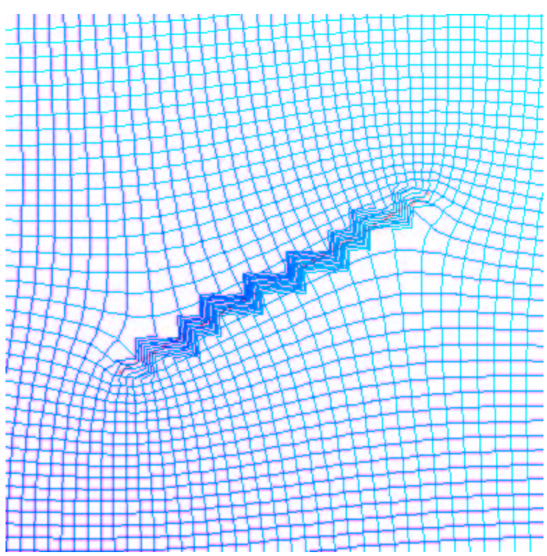
To compute flow past a plate with an angle of attack φ, we can rotate the plate into a horizontal position with $z \to e^{-i\varphi}z$, apply g^{-1} to make it into the unit disk, rotate back $z \to e^{i\varphi}z$ to restore the horizontal direction of overall flow, and apply g to squash the unit disk into a horizontal plate.

We see that the composition of maps $z \to g(e^{i\varphi}g^{-1}(e^{-i\varphi}z))$ takes the inclined plate to a horizontal one.

Thus, the complex potential for the original problem is $f(g(e^{i\varphi}g^{-1}(e^{-i\varphi}z)))$.

To compute g^{-1} we solve $g(z) = w$ for z as follows: $z + 1/z = w \Rightarrow z^2 + 1 = wz \Rightarrow z = \frac{1}{2}\left(w + \sqrt{w-2}\sqrt{w+2}\right)$, where we use the prinicpal branch of square root.

Here is a contour plot of the real and imaginary parts of the potential with $\varphi = \pi/6$.



Pictures generated with Maple: http://math.utsa.edu/~gokhman/courses/maple/flow_cyl.mws