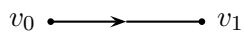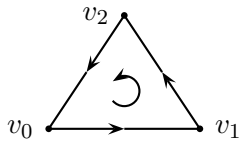**Simplices and boundary:**

Boundary homomorphism $\partial \colon \Delta_k \to \Delta_{k-1}$ is defined on a k simplex σ by $\partial(\sigma) = \sum\limits_{i=0}^{k} (-1)^i \sigma\big|_{[v_0,\dots \hat{v}_i,\dots v_k]}$

$\partial[v_0, v_1] = [v_1] - [v_0]$

$\partial[v_0, v_1, v_2] = [v_1, v_2] - [v_0, v_2] + [v_0, v_1]$
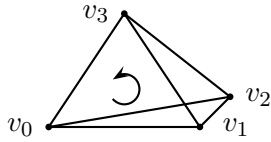
$\partial[v_0, v_1, v_2, v_3] = [v_1, v_2, v_3] - [v_0, v_2, v_3] + [v_0, v_1, v_3] - [v_0, v_1, v_2]$

Reference:

A. Hatcher, *Algebraic topology*, 2000